The Propagation of Light

(2/9, 2/14, 2/16)

- We start with reflection and refraction from the perspectives of Scattering Theory.
- Electromagnetic Theory approach provides a more complete description about the incident, reflected, and transmitted radiant flux densities (i.e., I_i, I_r, I_t, respectively).

HW 3
Hecht 4.19, 4.20, 4.22, 4.23, 4.24, 4.27, 4.28, 4.29
Due: 2/16/2023 by 11:30am

HW 4
Hecht 4.42, 4.44, 4.49, 4.50, 4.54, 4.55, 4.56, 4.58
Due: 2/21/2023 by 11:30am

Scattering Theory Approach

Reflection

- When a beam of light strikes an interface of two different transparent media (such as air and glass), some light is always scattered backward, and we call this phenomenon **reflection**.
- Imagine that light is traveling across a large homogeneous block of glass (Fig. 4.12a).
- Now, suppose that the block is sheared in half perpendicular to the beam. The two segments are then separated, exposing the smooth, flat surfaces depicted in Fig. 4.12b.
- Beam-I reflects off the right-hand block, and because light was initially traveling from a less to a more optically dense medium, this is called **external reflection**.
- Beam-II is reflected at the glass-air interface. The reflection is from a more to a less optically dense medium. This process is referred to as **internal reflection**.

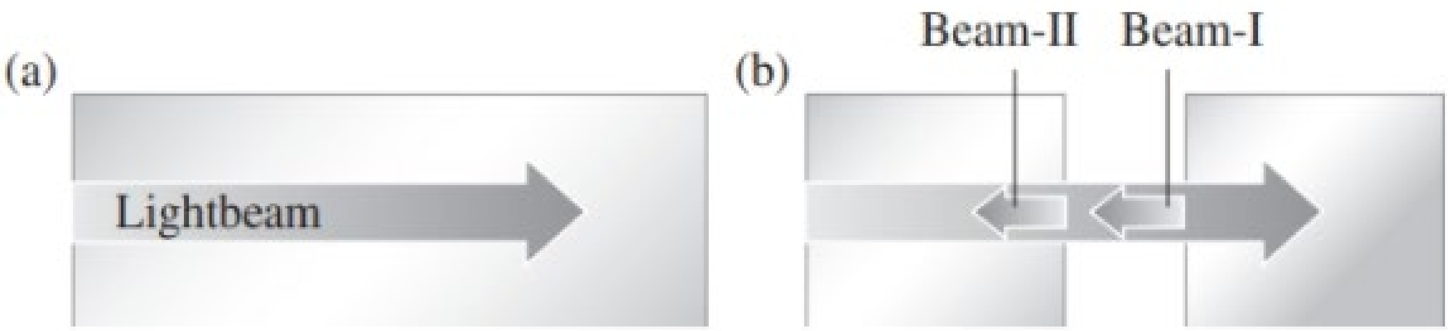


Figure 4.12 (a) A lightbeam propagating through a dense homogeneous medium such as glass. **(b)** when the block of glass is cut and parted, the light is reflected backward at the two new interfaces. Beam-I is externally reflected, and beam-II is internally reflected. Ideally, when the two pieces are pressed back together, the two reflected beams cancel one another.

- If the two glass regions are made to approach one another increasingly closely, the reflected light will diminish until it ultimately vanishes as the two faces merge and disappear and the block becomes continuous again.
- In other words, beam-I cancels beam-II; they must have been 180° out-of-phase. Remember this 180° relative phase shift between internally and externally reflected light (we will come back to it later on).

Snell's Law

- A **ray** is a line drawn in space corresponding to the direction of flow of radiant energy. *It is a mathematical construct and not a physical entity*.
- A ray representation is shown in Fig. 4.21 wherein all the angles are measured from the perpendicular (surface normal).
- The incident, reflected, and refracted rays all lie in the plane-of-incidence.
- In other words, the respective unit propagation vectors $\hat{k}_i$, $\hat{k}_r$, and $\hat{k}_t$ are coplanar.

Snell's Law

$$\theta_i = \theta_r \qquad (4.3)$$

$$n_i \sin\theta_i = n_t \sin\theta_t \qquad (4.4)$$

Note: n is the index of refraction.

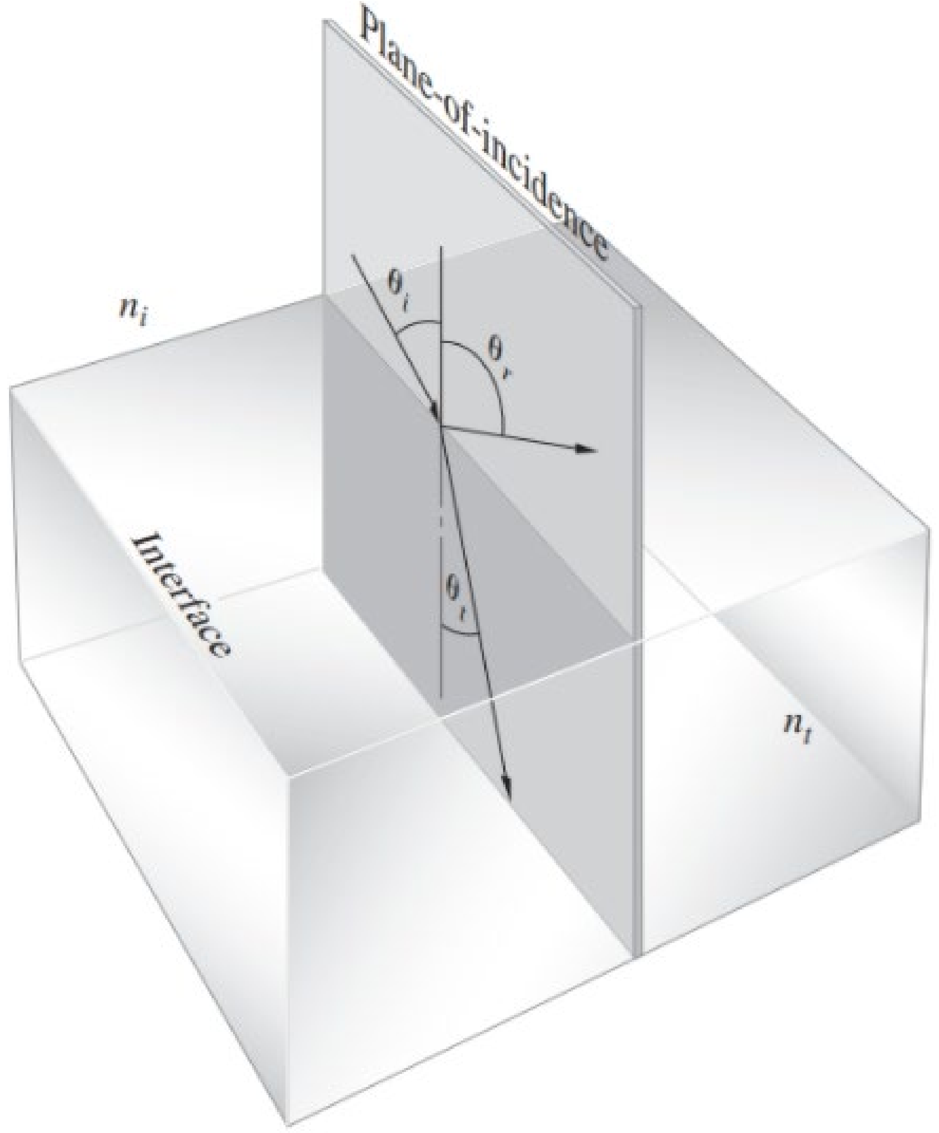


Fig. 4.21 The incident, reflected, and transmitted beams each lie in the plane-of-incidence.

Example 4.1 A ray of light in air having a specific frequency is incident on a sheet of glass. The glass has an index of refraction at that frequency of 1.52. If the transmitted ray makes an angle of 19.2° with the normal, find the angle at which the light impinges on the interface.

Solution

From Snell's Law

$$\sin\theta_i = \frac{n_t}{n_i}\sin\theta_t$$

$$\sin\theta_i = \frac{1.52}{1.00}\sin 19.2° = 0.4999$$

$$\text{and} \qquad \theta_i = 30°$$

- The ray entering a higher-index medium bends toward the normal (Fig. 4.23a). The reverse is also true (Fig. 4.23b); that is, on entering a medium having a lower index, the ray will bend away from the normal.
- Snell's Law can be rewritten in the form

$$\frac{\sin\theta_i}{\sin\theta_t} = n_{ti} \qquad (4.5)$$

where $n_{ti} \equiv n_t/n_i$ is the relative index of refraction of the two media. Note that $n_{ti} = v_i/v_t$; moreover, $n_{ti} = 1/n_{it}$.

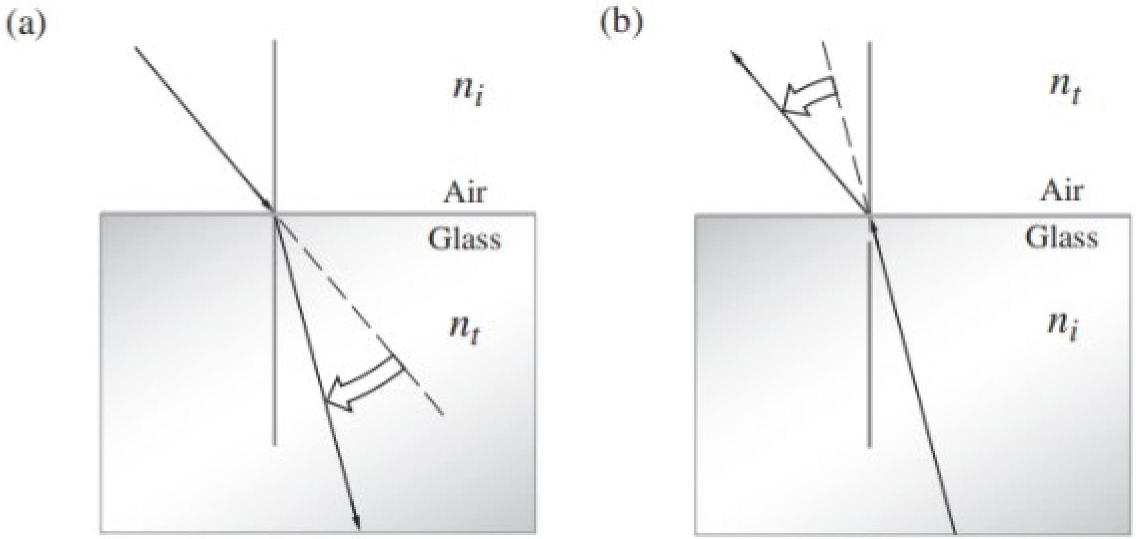


Fig. 4.23 The bending of rays at an interface.
(a) When a beam of light enters a more optically dense medium, one with a greater index of refraction ($n_i < n_t$), it bends toward the perpendicular.
(b) When a beam goes from a more dense to a less dense medium ($n_i > n_t$), it bends away from the perpendicular.

Example 4.2 A narrow laserbeam traveling in water having an index of 1.33 impinges at 40.0° with respect to the normal on a water–glass interface. If the glass has an index of 1.65

(a) determine the relative index of refraction.

(b) What is the beam's transmission angle in the glass

Total Internal Reflection

- Let's now take a closer look at the case of internal reflection ($n_i > n_t$).
- We allow θ_i to increase gradually, as indicated in Fig. 4.59.
- We know $\theta_t > \theta_i$, because

$$\frac{\sin\theta_i}{\sin\theta_t} = n_{ti} \qquad (4.5)$$

- When θ_t = 90°, sin θ_t = 1 and

$$\sin\theta_c = n_{ti} \qquad (4.69)$$

where θ_c is called **critical angle**, which is the special value of θ_i when θ_t = 90°.

- When θ_i greater than or equal to θ_c, all the incoming energy is reflected back into the incident medium. The process is known as **total internal reflection**.

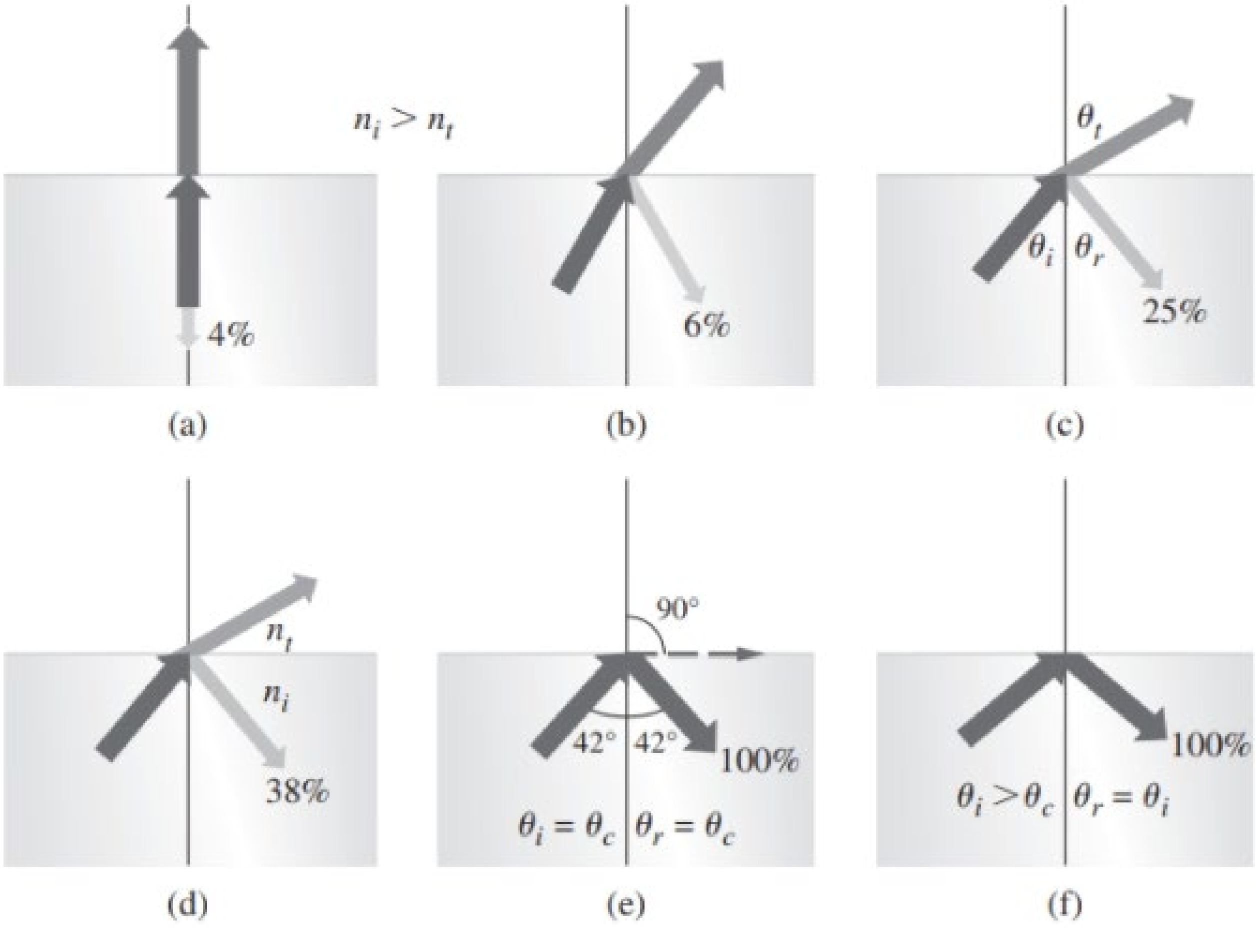


Fig. 4.59 Internal reflection and the critical angle.

- The critical angle for air–glass interface is roughly 42° (see Table 4.3).
- Consequently, a ray incident normally on the left face of either of the prisms in Fig. 4.60 will have a θ_i > 42° and therefore be internally reflected.
- This is a convenient way to reflect nearly 100% of the incident light.

TABLE 4.3 Critical Angles

n_{it}	θ_c (degrees)	θ_c (radians)	n_{it}	θ_c (degrees)	θ_c (radians)
1.30	50.2849	0.8776	1.50	41.8103	0.7297
1.31	49.7612	0.8685	1.51	41.4718	0.7238
1.32	49.2509	0.8596	1.52	41.1395	0.7180
1.33	48.7535	0.8509	1.53	40.8132	0.7123
1.34	48.2682	0.8424	1.54	40.4927	0.7067
1.35	47.7946	0.8342	1.55	40.1778	0.7012
1.36	47.3321	0.8261	1.56	39.8683	0.6958
1.37	46.8803	0.8182	1.57	39.5642	0.6905
1.38	46.4387	0.8105	1.58	39.2652	0.6853
1.39	46.0070	0.8030	1.59	38.9713	0.6802
1.40	45.5847	0.7956	1.60	38.6822	0.6751
1.41	45.1715	0.7884	1.61	38.3978	0.6702
1.42	44.7670	0.7813	1.62	38.1181	0.6653
1.43	44.3709	0.7744	1.63	37.8428	0.6605
1.44	43.9830	0.7676	1.64	37.5719	0.6558
1.45	43.6028	0.7610	1.65	37.3052	0.6511
1.46	43.2302	0.7545	1.66	37.0427	0.6465
1.47	42.8649	0.7481	1.67	36.7842	0.6420
1.48	42.5066	0.7419	1.68	36.5296	0.6376
1.49	42.1552	0.7357	1.69	36.2789	0.6332

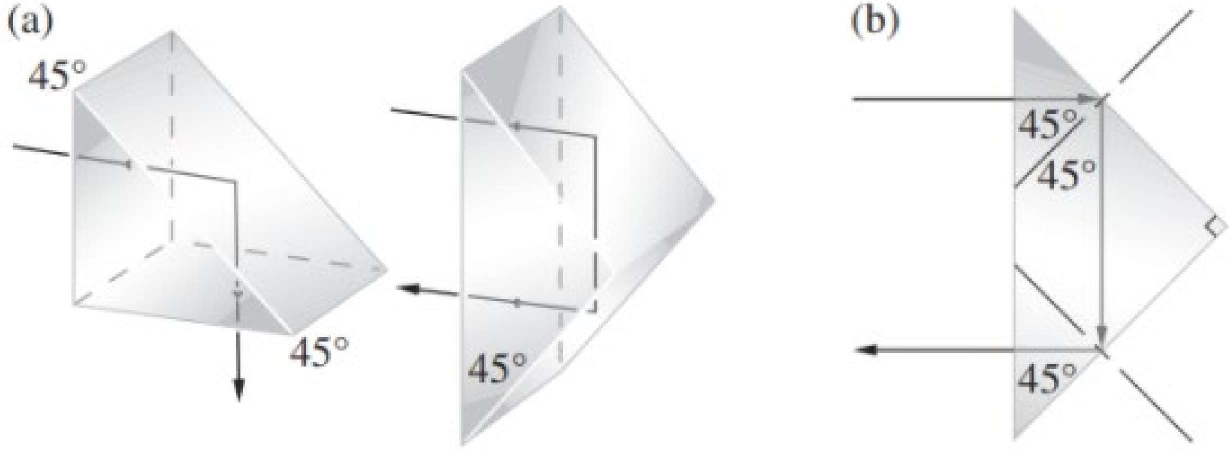
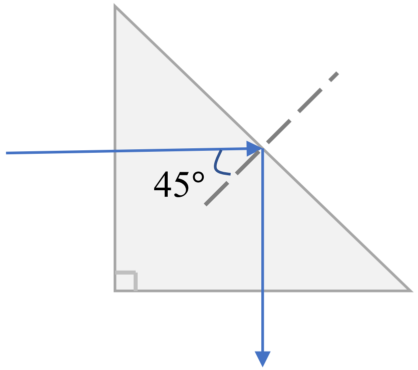


Fig. 4.60 Total internal reflection.



Refraction of Light from a Point Source

- A luminous point S on the left sends out light, some of which arrives at the interface where it is refracted as shown in Fig. 4.27
- Consider a narrow cone the rays will refract only a little, being nearly normal to the interface, and then will indeed appear to come from a single point P.
- The locations S and P are said to be **conjugate points**.
- Using triangles SAO and PAO in Fig. 4.27b

$$s_o tan\theta_i = s_i tan\theta_t$$

- Because the ray cone is narrow, θ_i and θ_t are small and we can replace the tangents with sines, so Snell's Law yields

$$s_i/s_o = n_t/n_i$$

- **Example:** Look straight down (i.e., to the left in Fig. 4.27b) on a fish (where $n_t = 1$, $n_i = 4/3$, and $n_t/n_i = 3/4$), which is 4.0 m beneath the surface and it will appear to be only 3.0 m below.

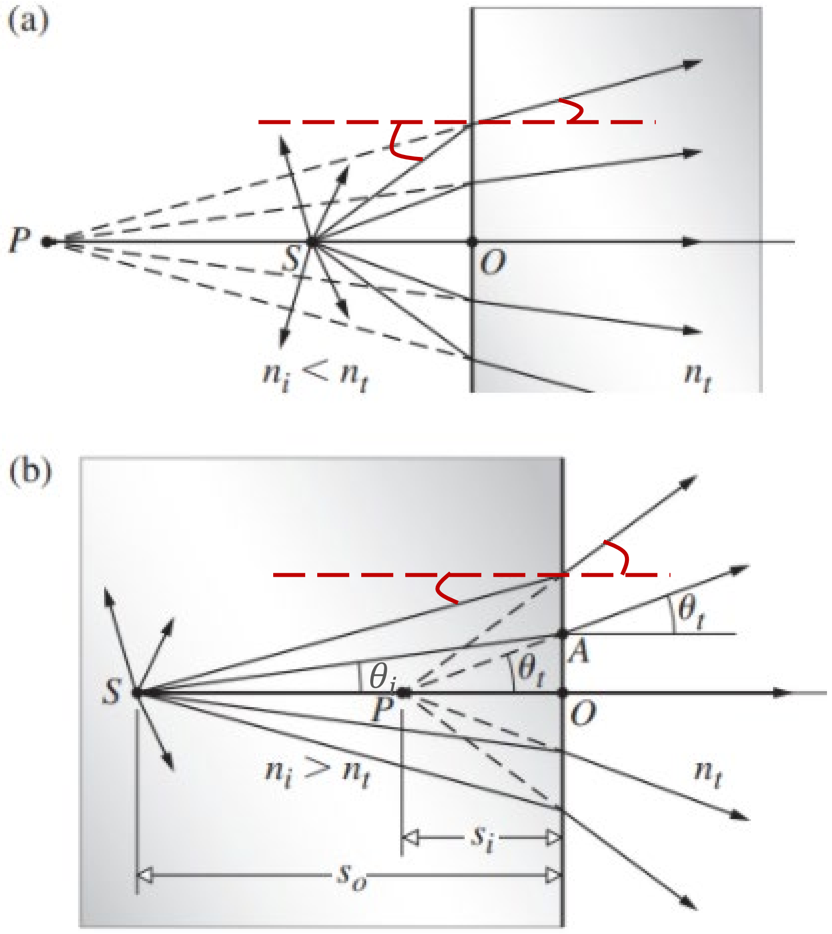


Fig. 4.27 The bending of light as it enters and leaves two different transparent materials across a planar interface. Now imagine that S in (b) is underwater—rotate the diagram 90° counterclockwise. An observer in the air would see S imaged at P.

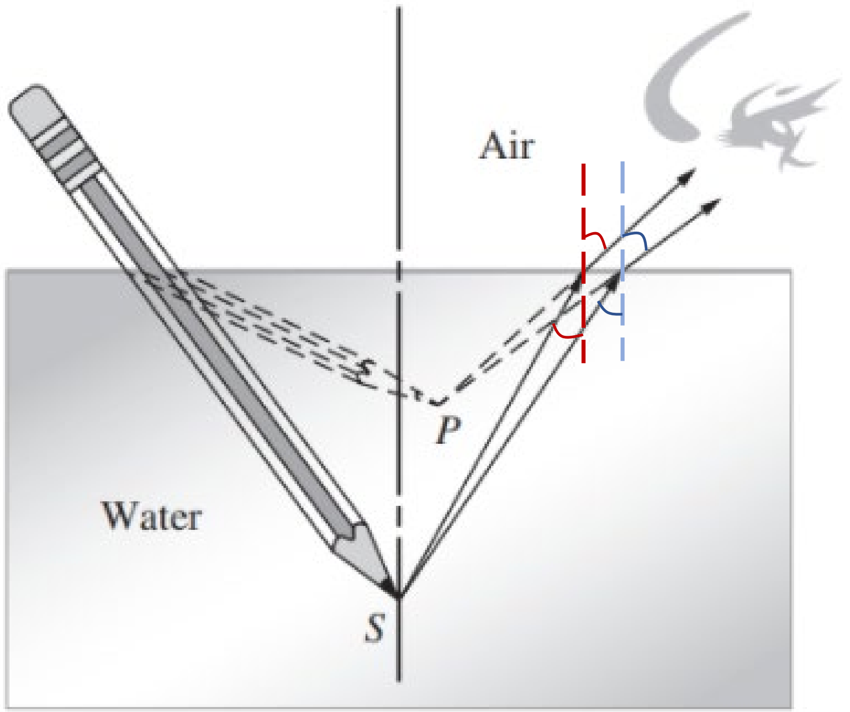
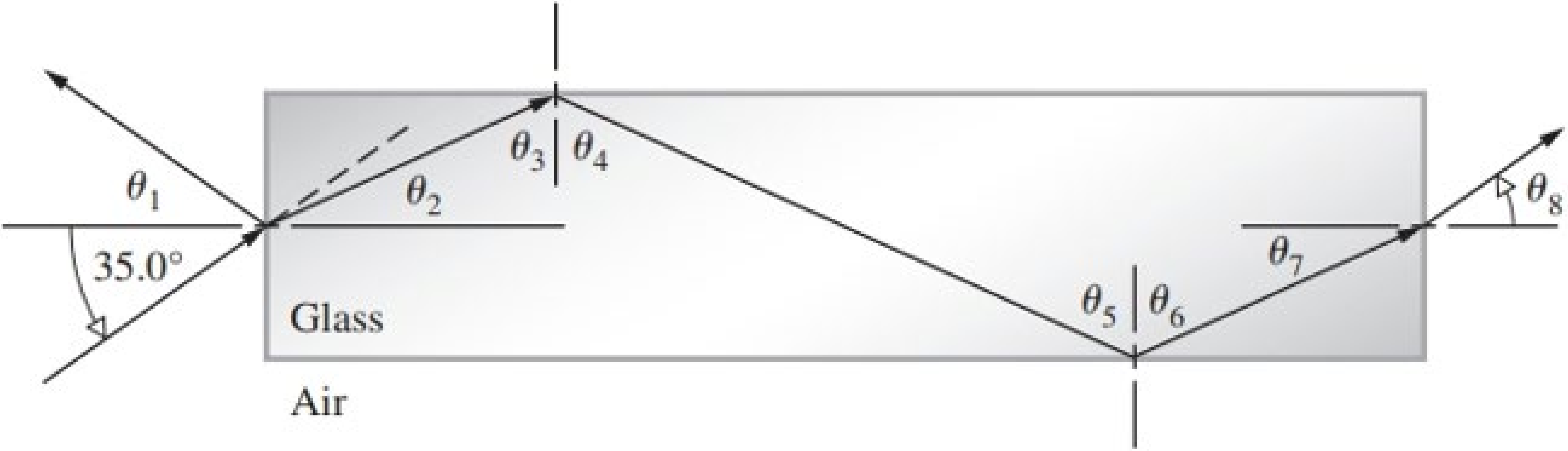


Fig. 4.29 Seeing an object beneath the surface of a quantity of water.

Rays from the submerged portion of the pencil bend on leaving the water as they rise toward the viewer. (E.H.)

Example 4.3 A ray impinges on a block of glass of index 1.55, as shown in the following illustration. Determine the angles θ_1, θ_2, θ_3, θ_4, θ_5, θ_6, θ_7, and θ_8.



Fermat's Principle

- **Fermat's Principle:** light, in going from point S to P, traverses the route having the smallest optical path length (OPL).
- $OPL = nl,$ where l is the spatial length and n is the index of refraction of the material.
- Fig. 4.35 depicts a point source S emitting a number of rays that are then "reflected" toward P.
- If we draw the rays as if they emitted from S′ (the image of S), none of the distances to P will have been altered (i.e., SAP = S′AP, SBP = S′BP, etc.).
- But obviously the straight-line path S′BP, which corresponds to $\theta_i = \theta_r$, is the shortest possible one.
- The same kind of reasoning (Problem 4.35) makes it evident that points S, B, and P must lie in what has previously been defined as the plane-of-incidence.

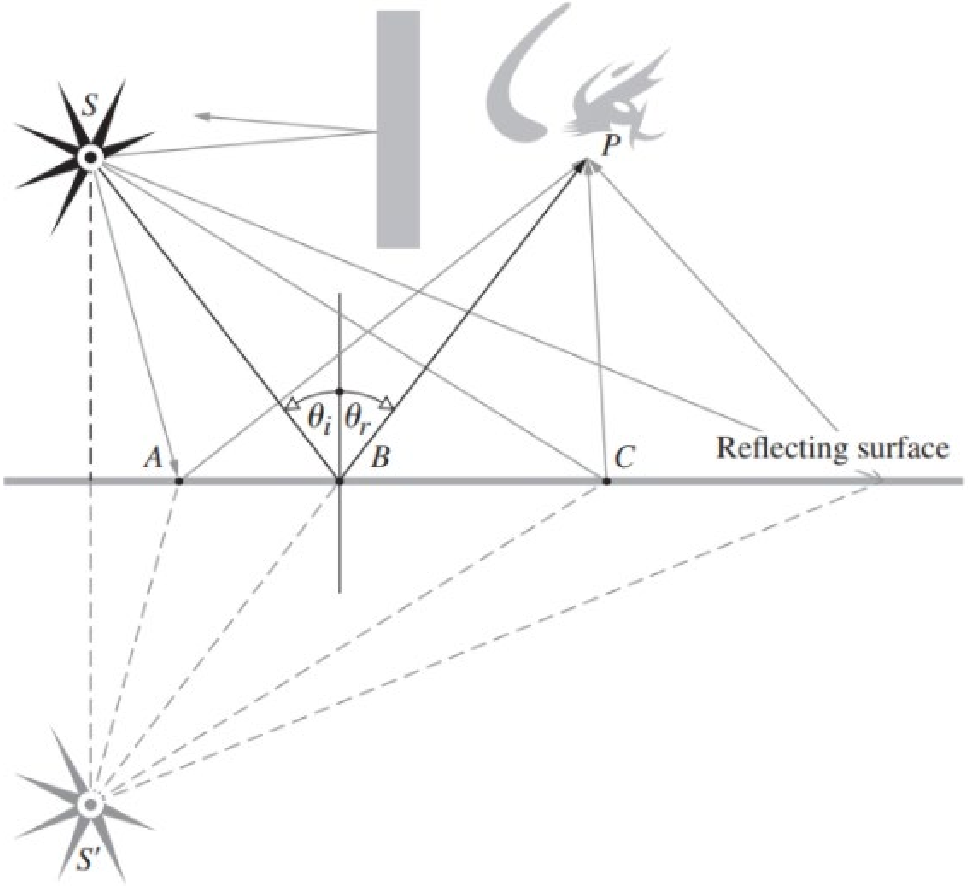


Fig. 4.35 Minimum path from the source S to the observer's eye at P.

- Fig. 4.36 depicts the application of Fermat's Principle to the case of refraction.
- We minimize t, the transit time from S to P, with respect to the variable x.
- The smallest transit time will then presumably coincide with the actual path. Hence

$$t = \frac{\overline{SO}}{v_i} + \frac{\overline{OP}}{v_t} \quad \Rightarrow \quad t = \frac{(h^2 + x^2)^{1/2}}{v_i} + \frac{[b^2 + (a-x)^2]^{1/2}}{v_t}$$

- To minimize $t(x)$ with respect to variations in x, we set $dt/dx = 0$, that is,

$$\frac{dt}{dx} = \frac{x}{v_i(h^2 + x^2)^{1/2}} + \frac{-(a-x)}{v_t[b^2 + (a-x)^2]^{1/2}} = 0$$

- Using the diagram, we can rewrite the expression as

$$\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_t}{v_t}$$

which is no less than Snell's Law (Eq. 4.4). If a beam of light is to advance from S to P in the least possible time, it must comply with the Law of Refraction.

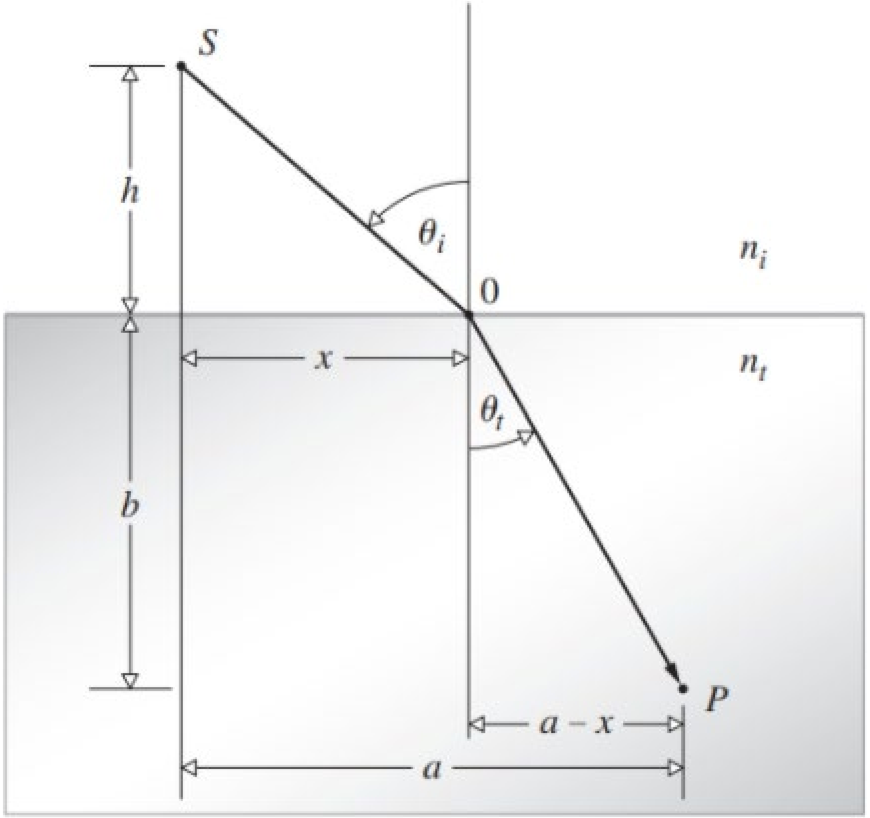


Fig. 4.36 Fermat's Principle applied to refraction.

The Electromagnetic Approach

Waves at an Interface

- Suppose that the incident monochromatic lightwave is planar, so that it has the form

$$\vec{\mathbf{E}}_i = \vec{\mathbf{E}}_{0i} \exp[i(\vec{\mathbf{k}}_i \cdot \vec{\mathbf{r}} - \omega_i t)] \qquad (4.11)$$

or

$$\vec{\mathbf{E}}_i = \vec{\mathbf{E}}_{0i} \cos(\vec{\mathbf{k}}_i \cdot \vec{\mathbf{r}} - \omega_i t) \qquad (4.12)$$

where the surfaces of constant phase are those for which $\vec{\boldsymbol{k}}_{\boldsymbol{i}} \cdot \vec{\boldsymbol{r}}$ = constant.

- Assume that $\vec{\boldsymbol{E}}_{0i}$ is constant in time, we can write the reflected and transmitted waves as

$$\vec{\mathbf{E}}_r = \vec{\mathbf{E}}_{0r} \cos(\vec{\mathbf{k}}_r \cdot \vec{\mathbf{r}} - \omega_r t + \varepsilon_r) \qquad (4.13)$$

and

$$\vec{\mathbf{E}}_t = \vec{\mathbf{E}}_{0t} \cos(\vec{\mathbf{k}}_t \cdot \vec{\mathbf{r}} - \omega_t t + \varepsilon_t) \qquad (4.14)$$

Here ε_r and ε_t are phase constants relative to $\vec{\boldsymbol{E}}_i$ and are introduced because the position of the origin is not unique.

The Fresnel Equations

- We now evaluate the interdependence shared by the amplitudes $\vec{E}_{0i}$, $\vec{E}_{0r}$, and $\vec{E}_{0t}$.
- suppose that a plane monochromatic wave is incident on the planar surface separating two isotropic media.
- We shall resolve its $\vec{E}$- and $\vec{B}$-fields into components parallel and perpendicular to the plane-of-incidence and treat these constituents separately.

Case 1: $\vec{E}$ perpendicular to the plane-of-incidence and $\vec{B}$ is parallel to it (Fig. 4.47).

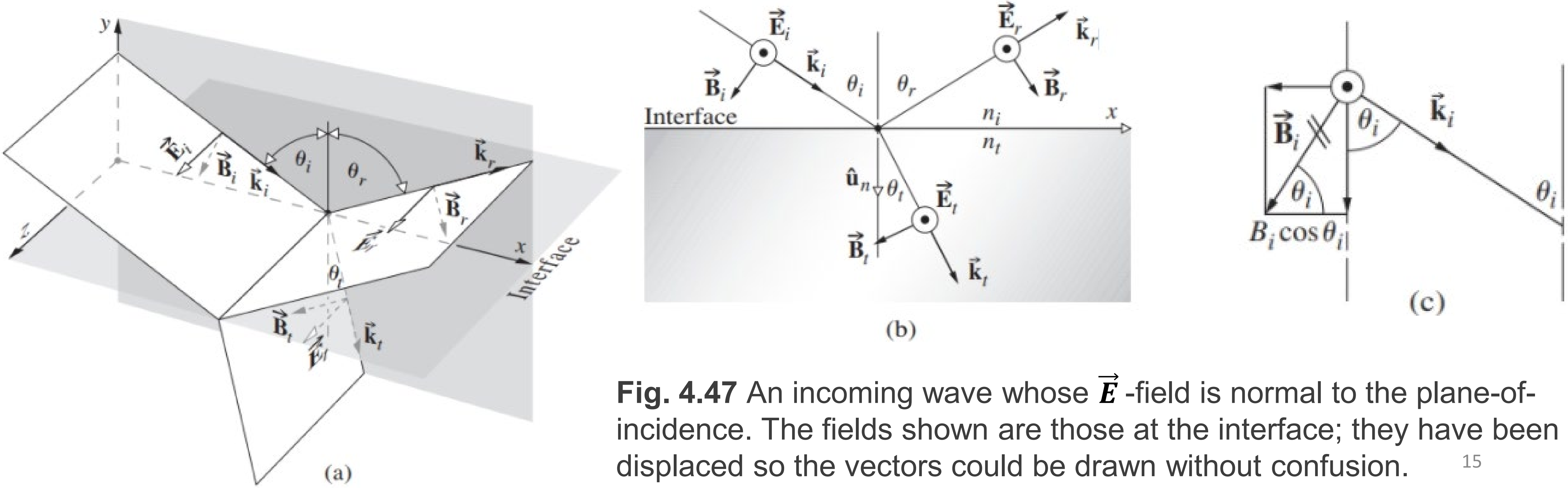


Fig. 4.47 An incoming wave whose $\vec{E}$ -field is normal to the plane-of-incidence. The fields shown are those at the interface; they have been displaced so the vectors could be drawn without confusion.

- Recall that $E = vB$, so that

$$\hat{\mathbf{k}} \times \vec{\mathbf{E}} = v\vec{\mathbf{B}} \qquad (4.23)$$

and

$$\hat{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \qquad (4.24)$$

- The laws of Electromagnetic Theory lead to certain requirements that must be met by the fields, and they are referred to as the **boundary conditions**.
- One of these is that the component of the electric field $\vec{\boldsymbol{E}}$ that is tangent to the interface must be continuous across it.
- Making use of the continuity of the tangential components of the $\vec{\boldsymbol{E}}$-field, we have at the boundary at <u>any time and any point</u>

$$\vec{\mathbf{E}}_{0i} + \vec{\mathbf{E}}_{0r} = \vec{\mathbf{E}}_{0t} \qquad (4.25)$$

- Another boundary condition is the continuity of the tangential component of $\vec{\boldsymbol{B}}/\mu$ that requires

$$-\frac{B_i}{\mu_i}\cos\theta_i + \frac{B_r}{\mu_i}\cos\theta_r = -\frac{B_t}{\mu_t}\cos\theta_t \qquad (4.26)$$

- From Eq. (4.23) we have

$$B_i = E_i/v_i \qquad (4.27) \qquad\qquad B_r = E_r/v_r \qquad (4.28) \qquad\qquad \text{and} \quad B_t = E_t/v_t \qquad (4.29)$$

- Since $v_i = v_r$ and $\theta_i = \theta_r$, Eq. (4.26) can be written as

$$\frac{1}{\mu_i v_i}(E_i - E_r)\cos\theta_i = \frac{1}{\mu_t v_t} E_t \cos\theta_t \qquad (4.30)$$

- Making use of Eqs. (4.12), (4.13), and (4.14) and remembering that the cosines equal one another at y = 0, we obtain

$$\frac{n_i}{\mu_i}(E_{0i} - E_{0r})\cos\theta_i = \frac{n_t}{\mu_t} E_{0t} \cos\theta_t \qquad (4.31)$$

- Combined with Eq. (4.25), this yields

$$\left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{\dfrac{n_i}{\mu_i}\cos\theta_i - \dfrac{n_t}{\mu_t}\cos\theta_t}{\dfrac{n_i}{\mu_i}\cos\theta_i + \dfrac{n_t}{\mu_t}\cos\theta_t} \qquad (4.32) \quad \text{and} \quad \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2\dfrac{n_i}{\mu_i}\cos\theta_i}{\dfrac{n_i}{\mu_i}\cos\theta_i + \dfrac{n_t}{\mu_t}\cos\theta_t} \qquad (4.33)$$

- The $\perp$ subscript means the case that $\vec{E}$ is perpendicular to the plane-of-incidence.
- These two expressions, which are completely general statements applying to any linear, isotropic, homogeneous media, are two of the **Fresnel Equations**.
- Most often one deals with dielectrics for which $\mu_i \approx \mu_t \approx \mu_0$; consequently, the common form of these equations is simply

$$r_\perp \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \quad (4.34)$$

$$t_\perp \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \quad (4.35)$$

- Here $r_\perp$ denotes the amplitude **reflection coefficient**, and $t_\perp$ is the amplitude **transmission coefficient**.

Case 2: $\vec{E}$ parallel to the plane-of-incidence.

- A similar pair of equations can be derived when the incoming $\vec{E}$-field lies in the plane-of-incidence as shown in Fig. 4.48.
- Continuity of the tangential components of $\vec{E}$ on either side of the boundary leads to

$$E_{0i}\cos\theta_i - E_{0r}\cos\theta_r = E_{0t}\cos\theta_t \qquad (4.36)$$

- In much the same way as before continuity of the tangential components of $\vec{B}/\mu$ yields

$$\frac{1}{\mu_i v_i}E_{0i} + \frac{1}{\mu_r v_r}E_{0r} = \frac{1}{\mu_t v_t}E_{0t} \qquad (4.37)$$

- Using the fact that $\mu_i = \mu_r$ and $\theta_i = \theta_r$, we can combine these formulas to obtain two more of the **Fresnel Equations** (next slide):

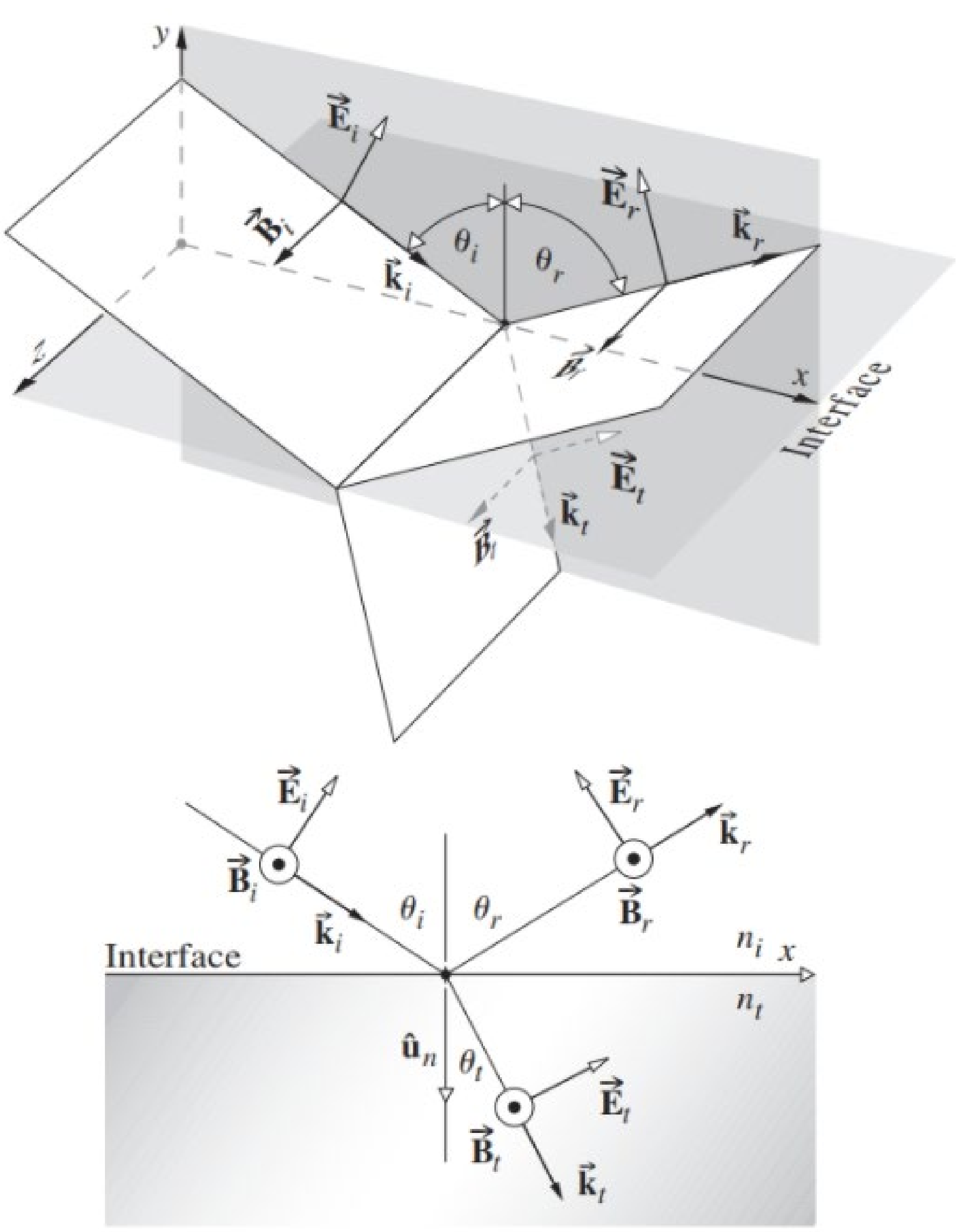


Fig. 4.48 An incoming wave whose $\vec{E}$-field is in the plane-of-incidence.

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \quad (4.40) \qquad \text{and} \qquad t_{\parallel} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \quad (4.41)$$

- One further notational simplification can be made using Snell's Law, at which the **Fresnel Equations** for dielectric media become (Problem 4.43)

$$r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad (4.42)$$

$$r_{\parallel} = +\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \quad (4.43)$$

$$t_{\perp} = +\frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \quad (4.44)$$

$$t_{\parallel} = +\frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} \quad (4.45)$$

Note: all possible sign variations have been labeled the Fresnel Equations. They must be related to the specific field directions from which they were derived.

Example 4.4 An electromagnetic wave having an amplitude of 1.0 V/m arrives at an angle of 30.0° to the normal in air on a glass plate of index 1.60. The wave's electric field is entirely perpendicular to the plane-of-incidence. Determine the amplitude of the reflected wave.

Solution

Since $(E_{0r})_\perp = r_\perp (E_{0i})_\perp = r_\perp (1\ \mathrm{V/m})$ we have to find

$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad [4.42]$$

But first we'll need θ_t, and so from Snell's Law

$$n_i \sin\theta_i = n_t \sin\theta_t \quad \Rightarrow \quad \sin\theta_t = \frac{n_i}{n_t}\sin\theta_i = \frac{1}{1.60}\sin 30.0^\circ = 0.3125 \quad \Rightarrow \quad \theta_t = 18.21^\circ$$

Hence $$r_\perp = -\frac{\sin(30.0^\circ - 18.2^\circ)}{\sin(30.0^\circ + 18.2^\circ)} = -\frac{\sin 11.8^\circ}{\sin 48.2^\circ} = -\frac{0.2045}{0.7455} = -0.274$$

and so $$(E_{0r})_\perp = r_\perp (E_{0i})_\perp = r_\perp (1.0\ \mathrm{V/m}) = -0.27\ \mathrm{V/m}$$

Interpretation of the Fresnel Equations

- This section examines the physical implications of the Fresnel Equations.
  - Determining the fractional amplitudes and flux densities that are reflected and refracted.
  - Determining the phase shifts that might be incurred in the process

Amplitude Coefficients

- At nearly normal incidence ($\theta_i \approx 0$), from Eqs. (4.34) and (4.40), we have

$$[r_{\parallel}]_{\theta_i=0} = [-r_{\perp}]_{\theta_i=0} = \left[\frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}\right]_{\theta_i=0} \quad (4.46)$$

- In the limit, as θ_i goes to 0, cos θ_i and cos θ_t both approach 1, and consequently

$$[r_{\parallel}]_{\theta_i=0} = [-r_{\perp}]_{\theta_i=0} = \frac{n_t - n_i}{n_t + n_i} \quad (4.47) \qquad [t_{\parallel}]_{\theta_i=0} = [t_{\perp}]_{\theta_i=0} = \frac{2n_i}{n_i + n_t} \quad (4.48)$$

- Thus, for example, at an air (n_i = 1)–glass (n_t = 1.5) interface at nearly normal incidence, the amplitude reflection coefficients equal ±0.2.

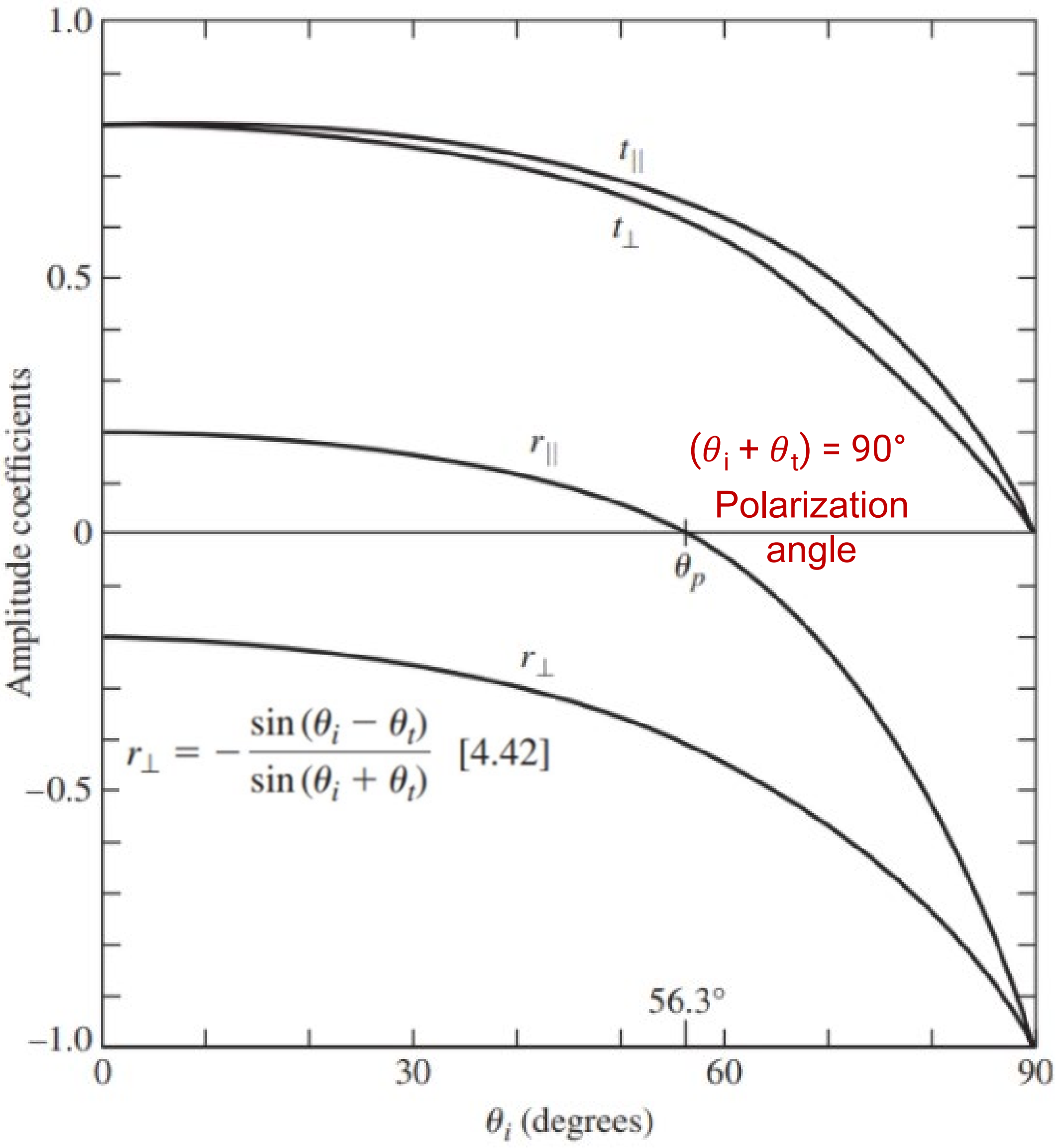


Fig. 4.49 The amplitude coefficients of reflection and transmission as a function of incident angle. These correspond to <u>external reflection</u> $n_t > n_i$ at an air–glass interface (n_{ti} = 1.5).

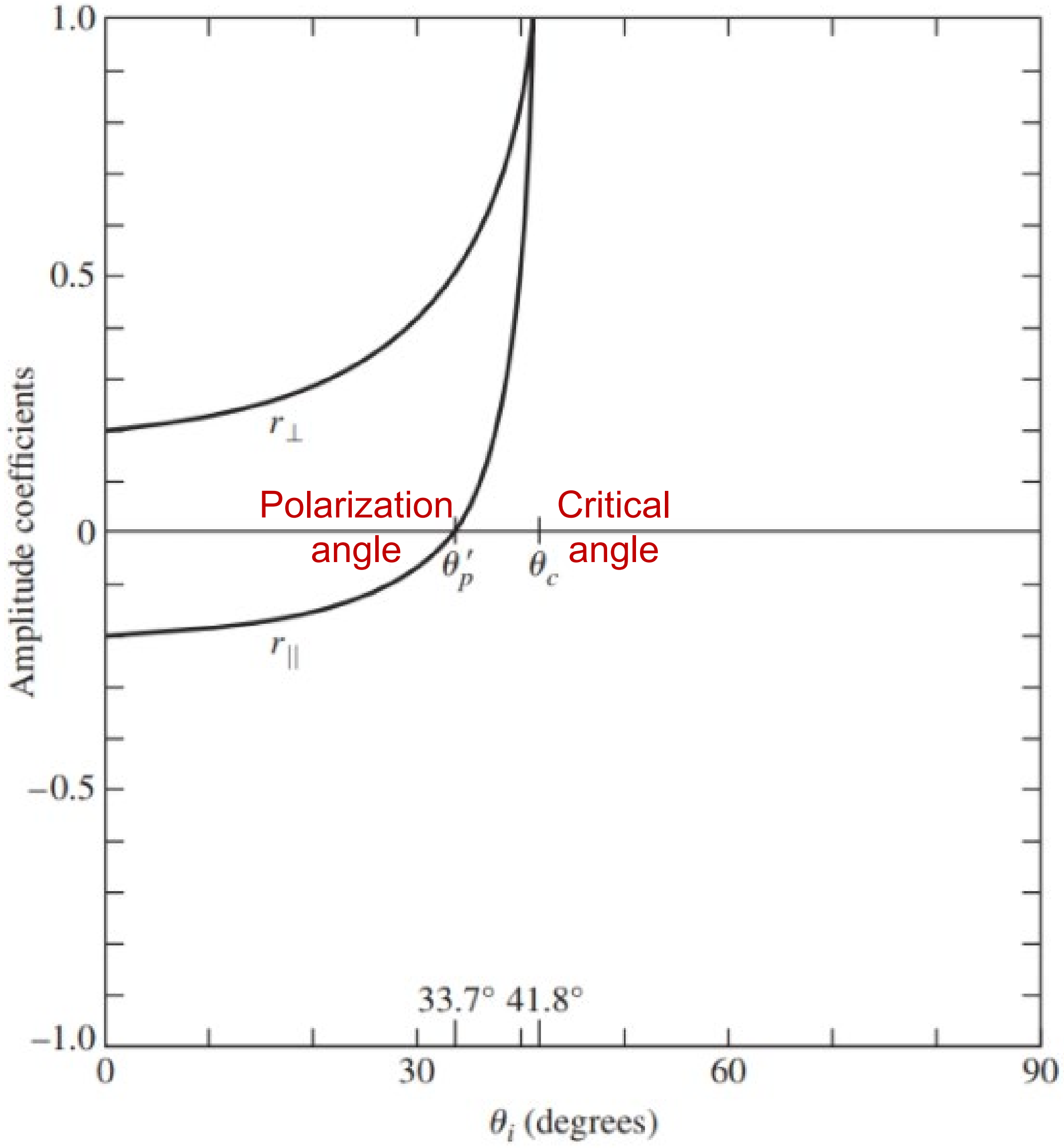


Fig. 4.50 The amplitude coefficients of reflection as a function of incident angle. These correspond to <u>internal reflection</u> $n_t < n_i$ at an air-glass interface (n_{ti} = 1/1.5).

Phase Shifts

- From Eq. (4.42) $r_\perp = -\dfrac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$ that $r_\perp$ is negative regardless of θ_i when $n_t > n_i$.
- The sign of $r_\perp$ is associated with the relative directions of $\left[\vec{\boldsymbol{E}}_{0i}\right]_\perp$ and $\left[\vec{\boldsymbol{E}}_{0r}\right]_\perp$.
- Thus, at the boundary $\left[\vec{\boldsymbol{E}}_i\right]_\perp$ and $\left[\vec{\boldsymbol{E}}_r\right]_\perp$ are <u>antiparallel</u> and therefore π out-of-phase with each other, as indicated by the negative value of $r_\perp$.
- Similarly, $t_\perp$ and $t_\parallel$ are always positive and the phase shift introduced $\Delta\varphi = 0$.
- Furthermore, when $n_i > n_t$ no phase shift in the normal component ($r_\perp$) results on reflection, that is, $\Delta\varphi_\perp = 0$ as long as $\theta_i < \theta_c$.
- When deal with $\left[\vec{\boldsymbol{E}}_i\right]_\parallel$, $\left[\vec{\boldsymbol{E}}_r\right]_\parallel$, and $\left[\vec{\boldsymbol{E}}_t\right]_\parallel$, it's necessary to define more explicitly what is meant by in-phase, since the field vectors are <u>coplanar</u> but generally not <u>colinear</u>.
- *We define that two fields in the incident plane are in-phase if their **y-components** are parallel, and are out-of-phase if the components are antiparallel.*
- With this definition we need only look at the vectors normal to the plane-of-incidence, whether they be $\vec{\boldsymbol{E}}$ or $\vec{\boldsymbol{B}}$, to determine the relative phase of the accompanying fields in the incident plane.
- When two $\vec{\boldsymbol{E}}$-fields are out-of-phase so too are their associated $\vec{\boldsymbol{B}}$-fields and vice versa.

- As an example, in Fig. 4.51a $\vec{\mathbf{E}}_\mathrm{i}$ and $\vec{\mathbf{E}}_\mathrm{t}$ are in-phase, as are $\vec{\mathbf{B}}_\mathrm{i}$ and $\vec{\mathbf{B}}_\mathrm{t}$, whereas $\vec{\mathbf{E}}_\mathrm{i}$ and $\vec{\mathbf{E}}_\mathrm{r}$ are out-of-phase, along $\vec{\mathbf{B}}_\mathrm{i}$ and $\vec{\mathbf{B}}_\mathrm{r}$.
- Similarly in Fig. 4.51b $\vec{\mathbf{E}}_\mathrm{i}$, $\vec{\mathbf{E}}_\mathrm{r}$, and $\vec{\mathbf{E}}_\mathrm{t}$ are in-phase, as are $\vec{\mathbf{B}}_\mathrm{i}$, $\vec{\mathbf{B}}_\mathrm{r}$, and $\vec{\mathbf{B}}_\mathrm{t}$.
- Now, the amplitude reflection coefficient for the parallel component is given by

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

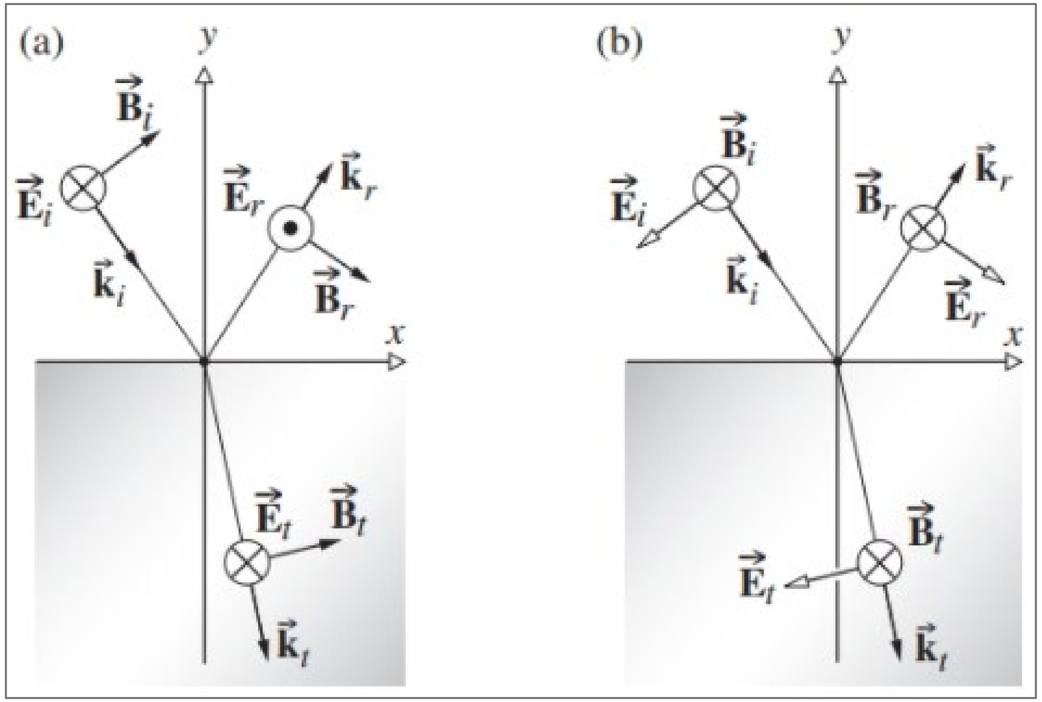
which is positive ($\triangle\,\varphi_\parallel = 0$) as long as

$$n_t \cos\theta_i - n_i \cos\theta_t > 0$$

that is, if $\sin\theta_i \cos\theta_i - \cos\theta_t \sin\theta_t > 0$

or equivalently $\sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t) > 0 \quad (4.51)$

Fig. 4.51 Field orientations and phase shifts.



- This will be the case for $n_i < n_t$ if

$$(\theta_i + \theta_t) < \pi/2 \quad (4.52)$$

and for $n_i > n_t$ when

$$(\theta_i + \theta_t) > \pi/2 \quad (4.53)$$

- Figure 4.52 (next slide) summarizes these conclusions.

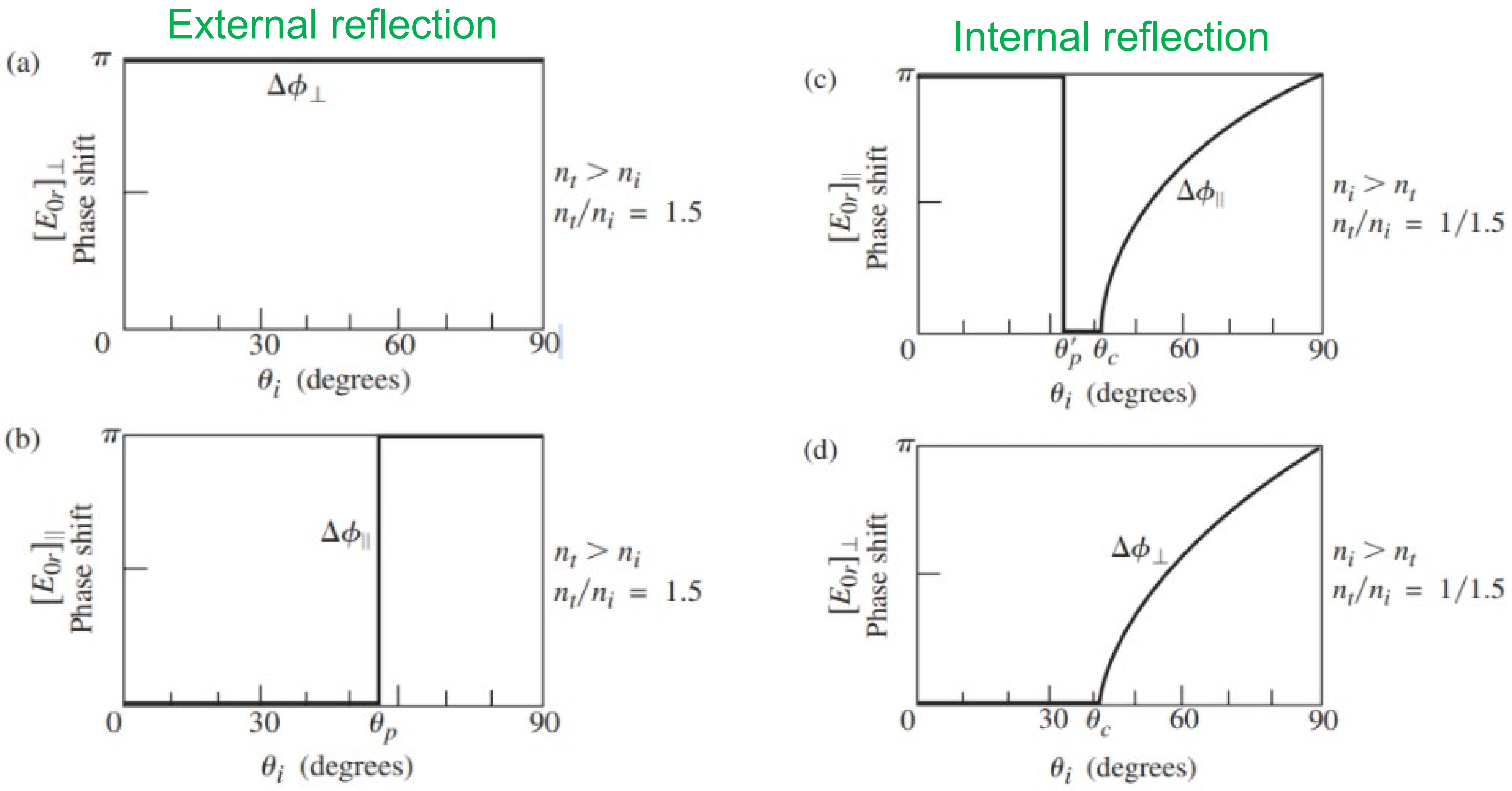


Fig. 4.52 Phase shifts for the parallel and perpendicular components of the $\vec{\boldsymbol{E}}$-field corresponding to internal and external reflection.

Reflectance and Transmittance

- Consider a circular beam of light incident on a surface, as shown in Fig. 4.55, such that there is an illuminated spot of area A.
- Recall that the power per unit area crossing a surface in vacuum whose normal is parallel to $\vec{S}$, the Poynting vector, is given by $\vec{\mathbf{S}} = c^2\epsilon_0\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ [3.40]
- Furthermore, the radiant flux density (W/m^2) or irradiance is

$$I = \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2 \qquad [3.44]$$

This is the average energy per unit time crossing a unit area normal to $\vec{S}$ (in isotropic media $\vec{S}$ is parallel to $\vec{k}$).

- let I_i, I_r, and I_t be the incident, reflected, and transmitted flux densities, respectively.
- The cross-sectional areas of the incident, reflected, and transmitted beams are, respectively, $A\cos\theta_i$, $A\cos\theta_r$, and $A\cos\theta_t$.

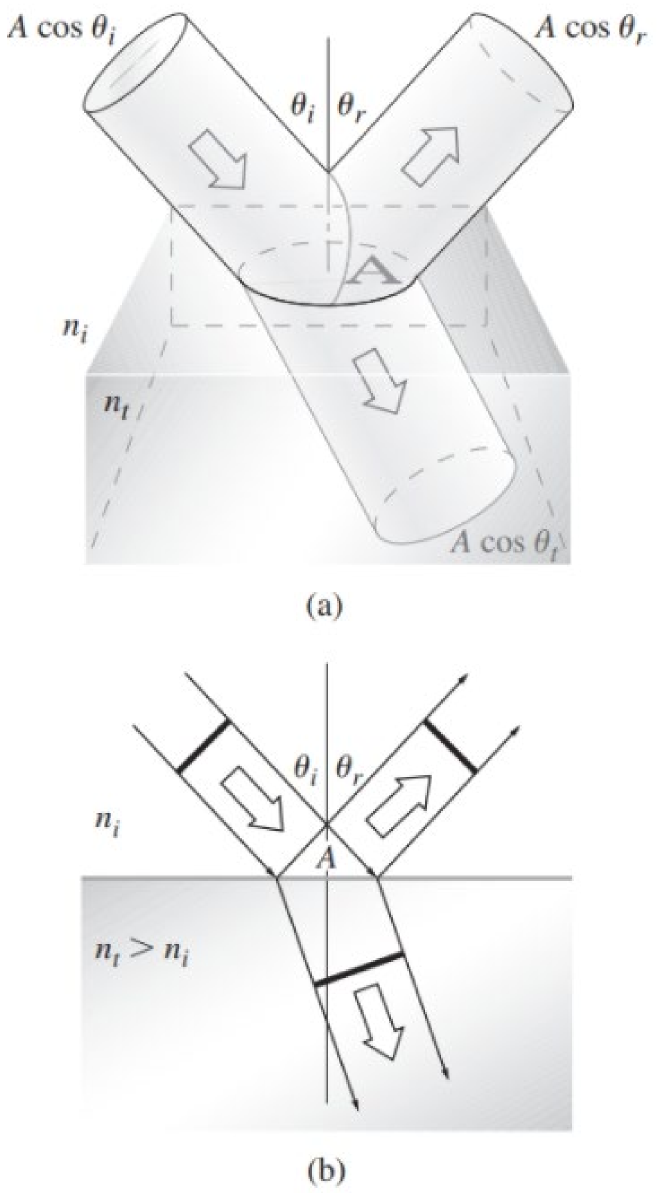


Fig. 4.55 Reflection and transmission of an incident beam.

- Accordingly, the incident power is $I_i A \cos\theta_i$; this is the energy per unit time flowing in the incident beam, and it's therefore the power arriving on the surface over *A*.
- Similarly, $I_r A \cos\theta_r$ is the power in the reflected beam, and $I_t A \cos\theta_t$ is the power being transmitted through A.
- We define the **reflectance R** to be the ratio of the reflected power (or flux) to the incident power

$$R \equiv \frac{I_r A \cos\theta_r}{I_i A \cos\theta_i} = \frac{I_r}{I_i} \qquad (4.54)$$

- In the same way, the **transmittance T** is defined as the ratio of the transmitted to the incident flux and is given by

$$T \equiv \frac{I_t \cos\theta_t}{I_i \cos\theta_i} \qquad (4.55)$$

- The quotient I_r/I_i equals $(v_r \epsilon_r E_{0r}^2/2)/\left(v_i \epsilon_i E_{0i}^2/2\right)$, and since the incident and reflected waves are in the same medium, $v_r = v_i, \epsilon_r = \epsilon_i$, and

$$\boxed{R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2 \qquad (4.56)}$$

- In like fashion (assuming $\mu_i = \mu_t = \mu_0$),

$$\boxed{T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)t^2 \quad (4.57)}$$

where $\mu_0\epsilon_t = 1/v_t^2$ and $\mu_0 v_t \epsilon_t = n_t/c$ was used.

- Observe that in Eq. (4.57) T is not simply equal to t^2, for two reasons.
  - First, the ratio of the indices of refraction must be there, since the speeds at which energy is transported into and out of the interface are different,
  - Second, the cross-sectional areas of the incident and refracted beams are different. The energy flow per unit area is affected accordingly, and that manifests itself in the presence of the ratio of the cosine terms.
- Based on the conservation of energy for the configuration depicted in Fig. 4.55, the total energy flowing into area A per unit time must equal the energy flowing outward from it per unit time:

$$I_i A \cos\theta_i = I_r A \cos\theta_r + I_t A \cos\theta_t \qquad (4.58)$$

- When both sides are multiplied by c, this expression becomes

$$n_i E_{0i}^2 \cos\theta_i = n_i E_{0r}^2 \cos\theta_i + n_t E_{0t}^2 \cos\theta_t$$

$$\text{or} \qquad 1 = \left(\frac{E_{0r}}{E_{0i}}\right)^2 + \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)\left(\frac{E_{0t}}{E_{0i}}\right)^2 \qquad (4.59)$$

this is simply $\qquad R + T = 1 \qquad (4.60)$

where there was no absorption.

- For ordinary "unpolarized" light, half oscillates parallel to the incident plane and half oscillates perpendicular to it. It follows from Eqs. (4.56) and (4.57) that

$$R_\perp = r_\perp^2 \qquad (4.61)$$

$$R_\parallel = r_\parallel^2 \qquad (4.62)$$

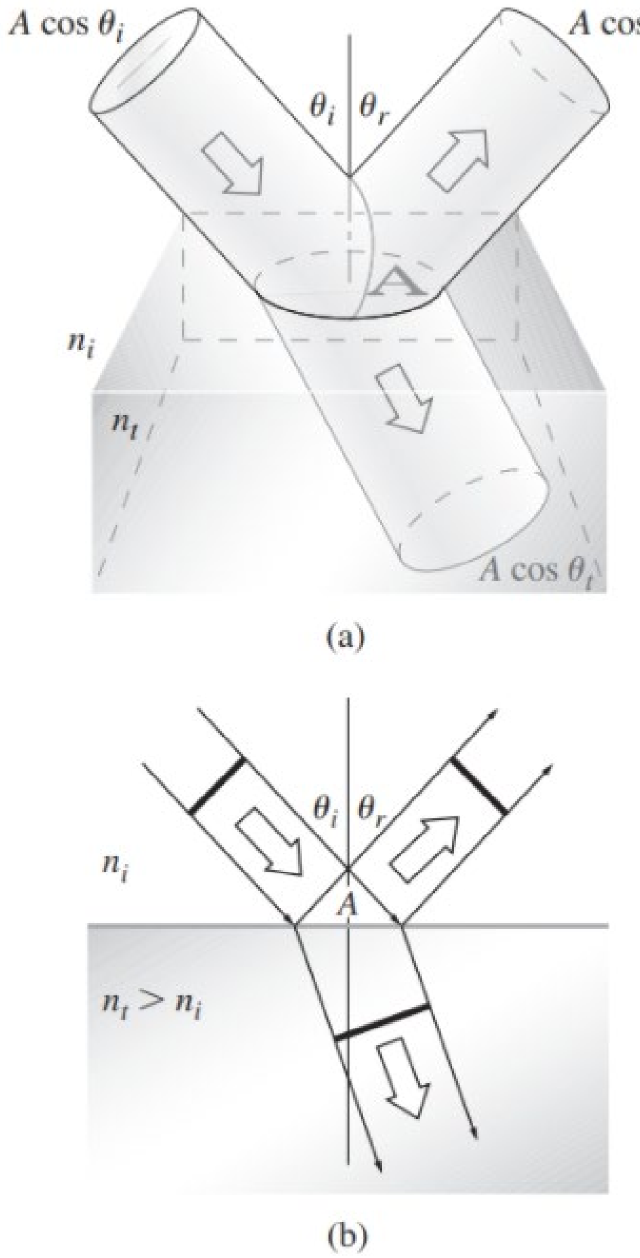


Fig. 4.55 Reflection and transmission of an incident beam.

$$T_\perp = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_\perp^2 \qquad (4.63) \qquad \text{and} \qquad T_\parallel = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_\parallel^2 \qquad (4.64)$$

which are illustrated in Fig. 4.56.

- Furthermore, it can be shown (Problem 4.73) that

$$R_\parallel + T_\parallel = 1 \qquad (4.65a)$$

and

$$R_\perp + T_\perp = 1 \qquad (4.65b)$$

- **Note:** $R_\perp$ is the fraction of $I_{i\perp}$ that is reflected, and not the fraction of I_i reflected. Accordingly, both $R_\perp$ and $R_\parallel$ can equal 1, and so the total reflectance for natural light is given by

$$R = \tfrac{1}{2}(R_\parallel + R_\perp) \qquad (4.66)$$

For a rigorous proof of this equation see Section 8.6.1.

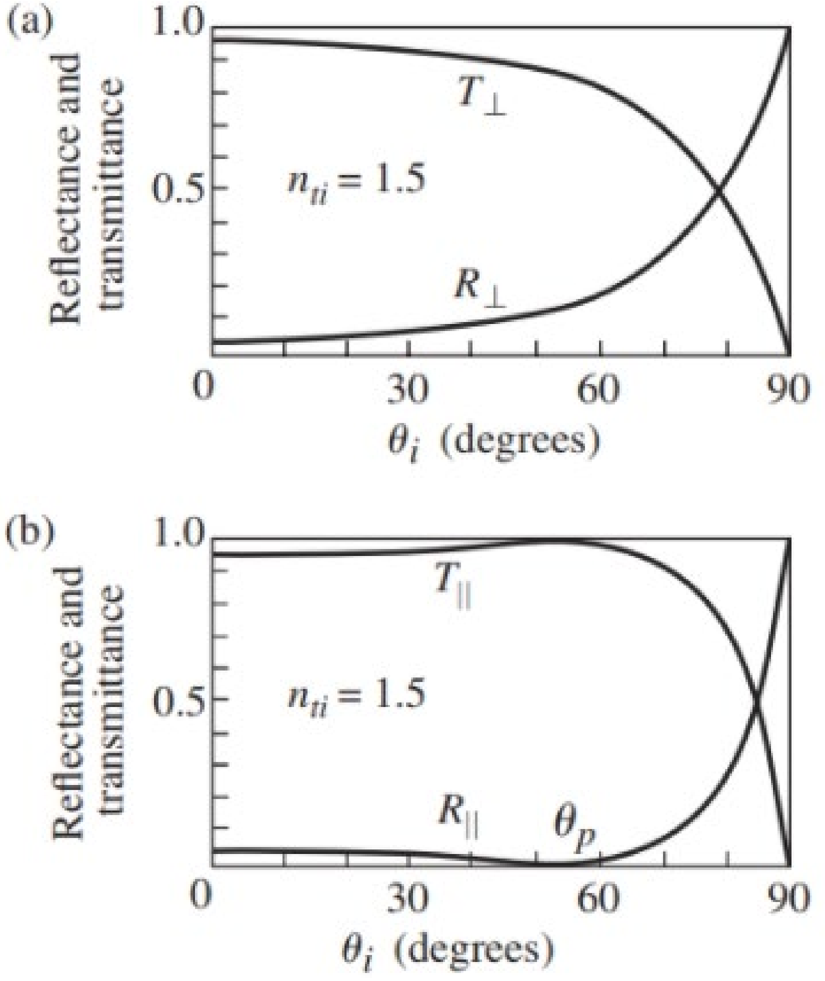


Fig. 4.56 Reflectance and transmittance versus incident angle.

Example 4.5 Light impinges on a slab of glass in air at the polarization angle θ_p. Assume that the net transmittance is known to be 0.86, and the incoming light is unpolarized. **(a)** Determine the percent of the incident power that is reflected. **(b)** If 1000 W comes in, how much power is transmitted with its E-field perpendicular to the plane-of-incidence?

Solution

(a) We are given that T = 0.86 and that since the beam is unpolarized half the light is perpendicular to the plane of incidence and half is parallel to it. For unpolarized light

$$T = \frac{1}{2}(T_{\parallel} + T_{\perp})$$

- Here θ_i = θ_p and so from Fig. 4.56 $T_{\parallel}$ = 1.0; all the light whose electric field is parallel to the plane of incidence is transmitted. Hence

$T = \frac{1}{2}(1 + T_{\perp}) = 0.86$ and for the perpendicular light $T_{\perp} = 1.72 - 1 = 0.72$

Since $R_{\perp} + T_{\perp} = 1$ $\Rightarrow$ $R_{\perp} = 1 - T_{\perp} = 0.28$

and the net reflected fraction is $R = \frac{1}{2}(R_{\parallel} + R_{\perp}) = \frac{1}{2}R_{\perp} = 0.14 = 14\%$

(b) Given 1000 W incoming, half of that, or 500 W, is perpendicular to the incident plane. Of this 72% is transmitted, since $T_\perp = 0.72$. Hence the power transmitted with its E-field perpendicular to the plane-of-incidence is

$$0.72 \times 500\ \text{W} = 360\ \text{W}$$

- When $\theta_i = 0$, the incident plane becomes undefined, and any distinction between the parallel and perpendicular components of R and T vanishes.
- In this case Eqs. (4.61) through (4.64), along with (4.47) and (4.48), lead to

$$R = R_{\parallel} = R_{\perp} = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2 \qquad (4.67) \qquad \text{and} \qquad T = T_{\parallel} = T_{\perp} = \frac{4n_t n_i}{(n_t + n_i)^2} \qquad (4.68)$$

Thus 4% of the light incident normally on an air–glass ($n_g = 1.5$) interface will be reflected back, whether internally, $n_i > n_t$, or externally, $n_i < n_t$ (Problem 4.70).

- This will be of concern to anyone who is working with a complicated lens system, which might have 10 or 20 such air–glass boundaries.
- Figure 4.57 is a plot of the reflectance at a single interface, assuming normal incidence for various transmitting media in air.
- Figure 4.58 depicts the corresponding dependence of the transmittance at normal incidence on the number of interfaces and the index of the medium.
- This is why you can't see through a roll of "clear" smooth-surfaced plastic tape, and it's also why the many elements in a periscope must be coated with antireflection films (Section 9.9.2).

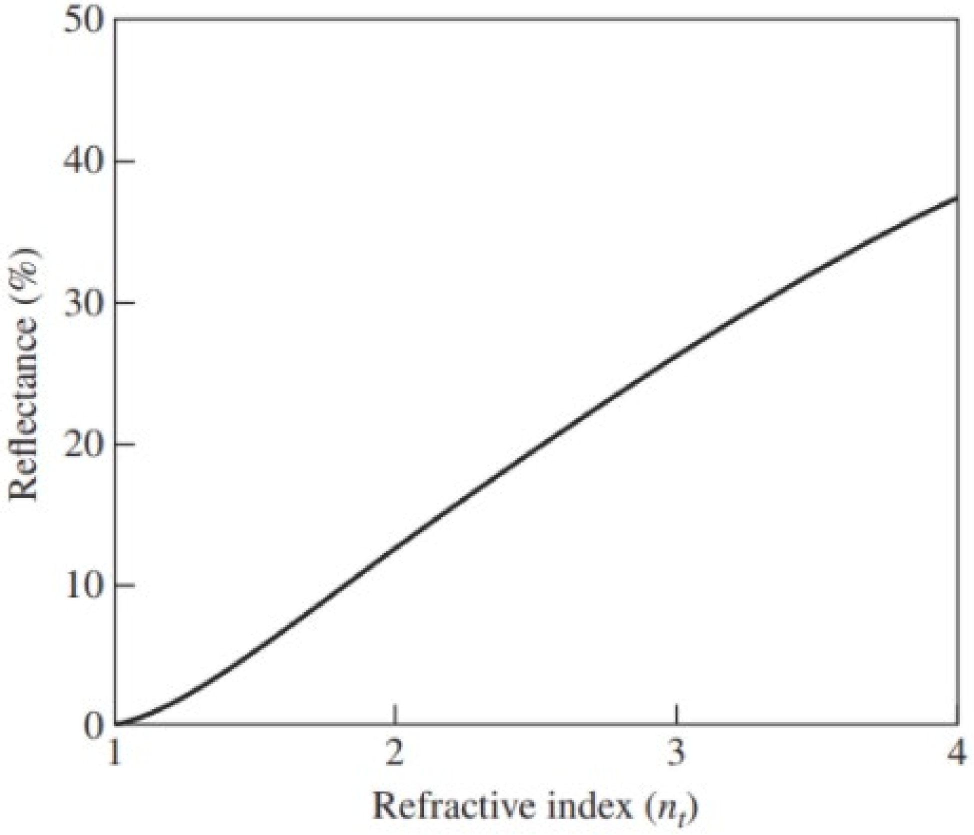


Fig. 4.57 Reflectance at normal incidence in air (n_i = 1.0) at a single interface.

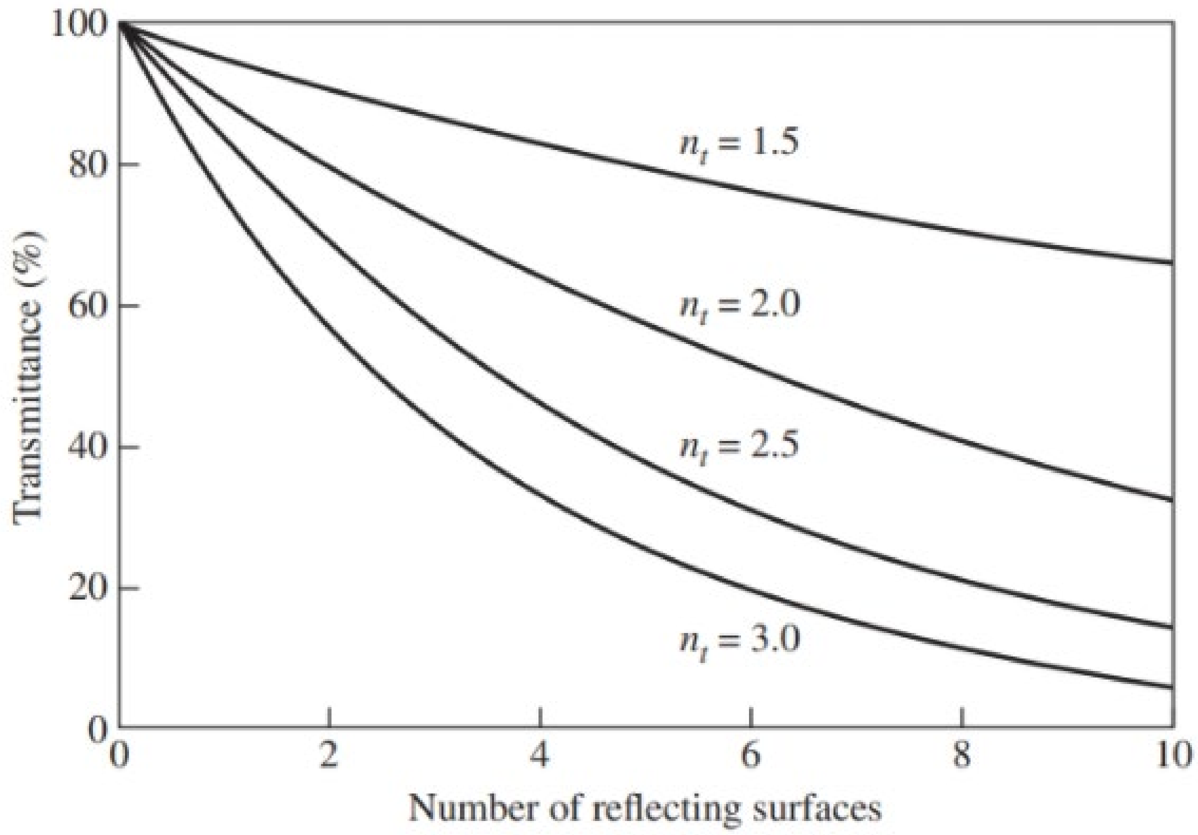


Fig. 4.58 Transmittance through a number of surfaces in air (n_i = 1.0) at normal incidence.

Example 4.6 Consider a beam of unpolarized light in air arriving at the flat surface of a glass sheet (n = 1.50) at the polarization angle θ_p. Considering Fig. 4.49 and the E-field oscillating parallel to the incident plane, determine $R_\parallel$ and then show by direct computation that $T_\parallel$ = 1.0. Since $r_\parallel$ = 0, why is $t_\parallel \neq 1$?

Solution

From Eq. (4.62) $R_\parallel = r_\parallel^2$ and $r_\parallel = 0$

hence $R_\parallel = 0$

and no light is reflected. On the other hand, from Eq. (4.64)

$$T_\parallel = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_\parallel^2$$

Using Fig. 4.49 and Eq. 4.41 $t_\parallel$ = 0.667 at $\theta_i = \theta_p$ = 56.3°,

and since $\theta_i + \theta_t$ = 90.0°,
θ_t = 33.7°, consequently $T_\parallel = \dfrac{1.5\cos 33.7°}{1.0\cos 56.3°}(0.667)^2 = 1.00$

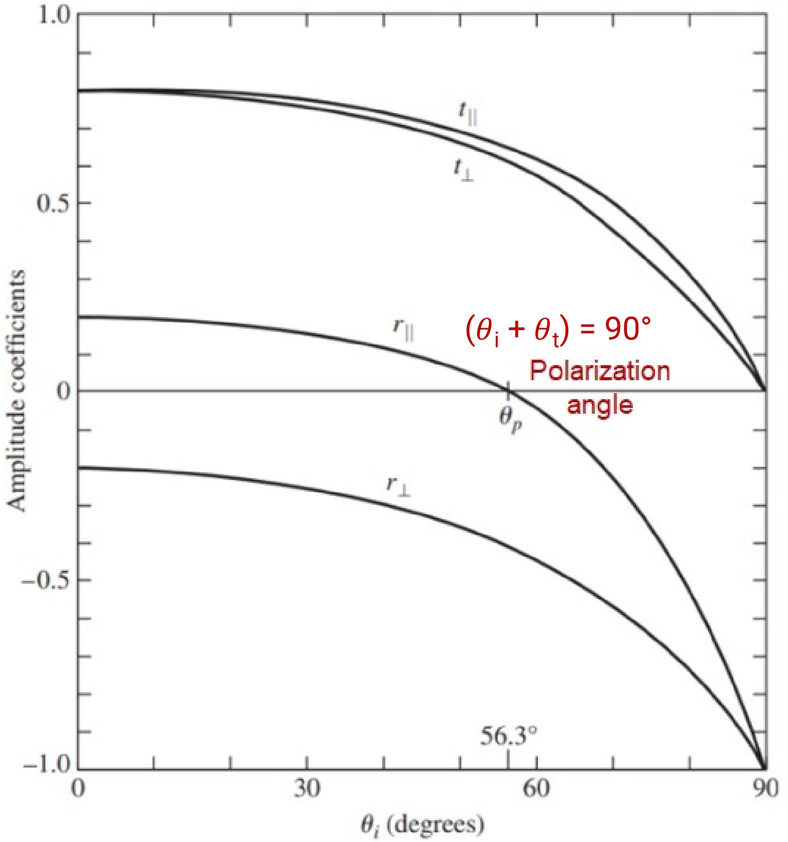


Fig. 4.49

- All the light is transmitted. Conservation of energy in a lossless medium tells us that $R_\parallel + T_\parallel = 1$; it does not say that $r_\parallel + t_\parallel = 1$.